Locker Problem – Answer Key

Name_____

One hundred students are assigned lockers 1 through 100. The student assigned to locker number 1 opens all 100 lockers. The student assigned to locker number 2 then closes all lockers whose numbers are multiples of 2. The student assigned to locker number 3 changes the status of all lockers whose numbers are multiples of 3 (e.g. locker number 3, which is open gets closed, locker number 6, which is closed, gets opened). The student assigned to locker number 4 changes the status of all locker whose numbers are multiples of 4, and so on for all 100 lockers.

1.   Which lockers will be left open?

    1, 4, 9, 16, 25, 36, 49, 64, 81, 100

2.   Explain how you determined that these particular lockers will be open.

    Either by looking at a pattern with the opened and closed lockers…open, 2 closed, open, 4 closed, open, 6 closed, open, 8 closed, open….

    Noticing that the lockers opened are perfect squares

    Other?

3. What do you notice about these particular locker numbers?

    The locker numbers are perfect squares.

4. Why are these specific locker numbers still open?

    They have an odd number of factors.

5. How many lockers, and which ones, were touched exactly twice? How do you know?

    25; 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79,83, 89, 97; they are the prime numbers which only have two factors.

6.   Which students touched both lockers 36 and 48? How do you know?

Students with lockers 1, 2, 3, 4, 6, and 12. These are the common factors of 36 and 48.

7. Which locker was switched the most times? How do you know?

72 and 96. They have the most factors.

Extension to Locker Problem

Based on the Locker Problem, how do you determine how many times a particular locker has been switched?

How can we determine how many factors a given number has without having to try list them all?

1.   How does the prime factorization and the number of factors a number has compare? To explore this, find the prime factorization of a number and find how many factors it has. Try this with numbers that only have a single prime factor such as 8 or 9.

Examples:
$8 = 2^3$ factors: 1, 2, 4, 8
$9 = 3^2$ factos: 1, 3, 9
$16 = 2^4$ factors: 1, 2, 4, 8, 16
$49 = 7^2$ factors: 1, 7, 49
The number of factors is 1 more than the exponent on the prime number in the prime factorization.

2.   Now, how can you use what you found in number 1 to determine how many factors a bigger number has? Try 24 using its prime factorization and see if you're right by listing the factors of 24.

$24 = 2^3 \cdot 3$ 2^3 has four factors 3^1 has two factors
So, 24 has $4 \cdot 2 = 8$ factors
Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24 = 8 factors

3.   How many factors does the number 432 have?

$432 = 2^4 \cdot 3^3$
$5 \cdot 4 = 20$ factors

4. How can the prime factorization now be used to find the factors of the number? Try a number like 12 or 24 to help you get started.

$12 = 2^2 \cdot 3$
Take each factor of 2^2 and multiply it by each factor of 3. Don't forget that each also has a factor of 1 that isn't listed.
$1 \cdot 1 = 1$ $2^1 \cdot 1 = 2$ $2^2 \cdot 1 = 4$ $2^1 \cdot 3^1 = 6$ $2^2 \cdot 3^1 = 12$
$1 \cdot 3^1 = 3$
Factors of 12 are 1, 2, 3, 4, 6, 12

5. List the factors of 432.

From #3, $432 = 2^4 \cdot 3^3$

$1 \cdot 1 = 1$	$2^1 \cdot 1 = 2$	$2^2 \cdot 1 = 4$	$2^3 \cdot 1 = 8$	$2^4 \cdot 1 = 16$	$2^1 \cdot 3^1 = 6$
$2^1 \cdot 3^2 = 18$	$2^1 \cdot 3^3 = 54$	$2^2 \cdot 3^1 = 12$	$2^2 \cdot 3^2 = 36$	$2^2 \cdot 3^3 = 108$	$2^3 \cdot 3^1 = 24$
$2^3 \cdot 3^2 = 72$	$2^3 \cdot 3^3 = 216$	$2^4 \cdot 3^1 = 48$	$2^4 \cdot 3^2 = 144$	$2^4 \cdot 3^3 = 432$	$1 \cdot 3^1 = 3$
$1 \cdot 3^2 = 9$	$1 \cdot 3^3 = 27$				

6. Going back to the Locker Problem, if there were 500 lockers instead of 100, how many times would locker number 432 be switched? Which students touched locker 432.

20 times; Students with locker numbers 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 36, 48, 52, 72, 108, 144, 216, 432